ON DEPENDENCE OF RESOLVING POWER OF FABRY-PEROT ETALON, LUMMER-GEHRCKE PLATE AND TRANSMISSION ECHELON ON STAGE OF RESOLUTION*

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(*Received for publication, July 31, 1954*)

ABSTRACT. The dependence of the resolving power on the stage of resolution desired and the detecting instrument available has been studied for the case of Fabry-Perot etalon, Lummer-Gehrcke plate and transmission echelon including the absorption by the material of the last two instruments. Tables and graphs have been given to show the dependence.

INTRODUCTION

Ditchburn (1930) has pointed out that the value of $I_{min.}/I_{max.}$ $(=c)$ of a spectral pattern at limiting resolution chosen for the calculation of resolving power of an optical instrument decides the stage of resolution desired by a detecting instrument. Using a microphotometer he was able to distinguish between three important stages of resolution:

Stage of resolution	$c = I_{min.}/I_{max.}$
(i) Detection of inhomogeneity in radiation	0.98
(ii) Approximate measurement of wavelength and relative intensities	0.8
(iii) Accurate measurement of wavelength and relative intensities	0.4

Sharma and Sodha (1954) have studied the variation of resolving power of grating, prism and reflecting echelon with c, which is characteristic of the stage of the resolution desired and the detecting instrument used. This paper presents a similar study for Fabry-Perot etalon, Lummer-Gehrcke plate and transmission echelon.

FABRY-PEROT ETALON

The intensity pattern in Fabry-Perot etalon is given by

$$I_1 = \frac{I_0}{1+F\sin^2\pi\,(n_0+n)} = \frac{I_0}{1+X^2}.$$

where, F is the coefficient of fineness, n_0+n is the order, n_0 being an integer and n a fraction and $X=\pi n F^{\frac{1}{2}}$.

The intensity pattern of another line separated by a small order Δn is given by

* Communicated by Dr. K. Majumdar

$$I_2 = \frac{I_0}{1+F\sin^2\pi(n_0+n-\Delta n)} = \frac{I_0}{1+(X-a)^2}$$

where $a=\pi\Delta n F^{1/2}$. The resultant intensity pattern is given by

$$I=I_1+I_2=I_0\left\{\frac{1}{1+X^2}+\frac{1}{1+(X-a)^2}\right\} \qquad \ldots \quad (1)$$

The maxima ($X\approx 0$ or a) and minimum ($X=a/2$) of the resultant intensity pattern are given by

$$\frac{I_{max}}{I_0}=1+\frac{1}{1+a^2}=\frac{2+a^2}{1+a^2} \qquad \ldots \quad (2)$$

and

$$\frac{I_{min}}{I_0}=\frac{2}{1+a^2/4}=\frac{8}{4+a^2} \qquad \ldots \quad (3)$$

Putting the criterion

$$I_{min.}/I_{max.}=c \qquad \ldots \quad (4)$$

for limiting resolution we get,

$$8(1+a^2)=c(4+a^2)(2+a^2) \qquad \ldots \quad (5)$$

Considering only the positive root of the above equation one gets

$$a=\left\{\frac{(4-3c)+\sqrt{(4-3c)^2+8c(1-c)}}{c}\right\}^{1/2} \qquad \ldots \quad (6)$$

The resolving power is given by :

$$\frac{\lambda}{d\lambda}=\frac{n_0}{\Delta n}=\frac{\pi}{a}n_0F^{1/2}=\alpha n_0F^{1/2} \qquad \ldots \quad (7)$$

Table I and figure 1 illustrate the variation of α with c. The values of α for $c=.8$ and $.981$ corresponding to Rayleigh and Abbe criteria are the same as given by Meissner (1941) and Sodha (1953a).

TABLE I

c	a^2	α
.40	14.815	.816
.45	12.556	.887
.50	10.745	.958
.55	9.253	1.033
.60	7.999	1.111
.65	6.929	1.193
.70	6.000	1.282
.75	5.182	1.380
.80	4.449	1.490
.85	3.785	1.614
.90	3.169	1.764
.95	2.584	1.954
.981	2.269	2.085

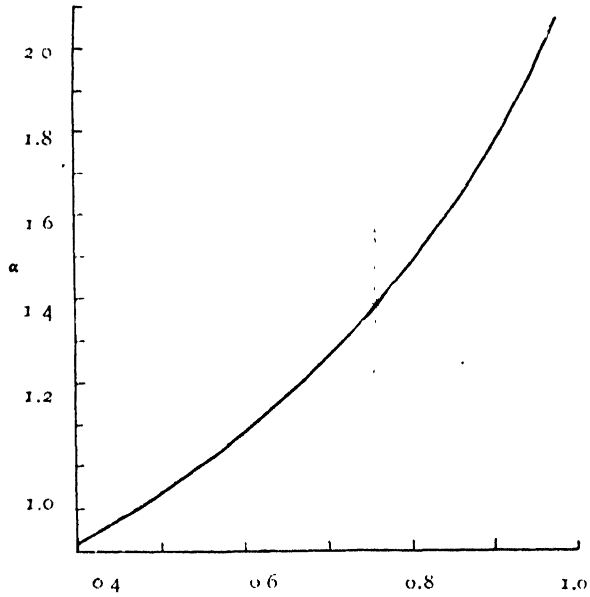

FIG. 1. Dependence of α on C

LUMMER-GEHRCKE PLATE

The expression for intensity of an emerging beam system of a Lummer-Gehrcke plate is given by (Gehrcke, 1906 and Candler, 1951).

$$I = I_0 \frac{\{(1-R^N)^2 + 4R^N \sin^2 N\varphi/2\}(1-R)}{(1-R)^2 + 4R \sin^2\varphi/2} \qquad \ldots \quad (8)$$

symbols having their usual meanings. Putting $F = 4R/(1-R)^2$ and $F_N = 4R^N/(1-R^N)^2$, equation (8) is simplified to

$$\frac{I}{I_0} = \frac{(1-R^N)^2}{(1-R)} \cdot \frac{1+F_N \sin^2 N\varphi/2}{1+F \sin^2\varphi/2} \qquad \ldots \quad (8a)$$

The intensity I_c at the centre of the fringe ($\varphi = 0$) is given by

$$I_c = I_0 \frac{(1-R^N)^2}{1-R} \qquad \ldots \quad (9)$$

If I_1 and I_2 represent the intensities at φ and 2φ then the ratio $I_{min.}/I_{max.}$ is given by

$$I_{min.}/I_{max} = \frac{2I_1}{I_c + I_2} \qquad \ldots \quad (10)$$

Combining equations (10), (9), (8a) and (4) we get

$$1+\frac{1+F_N \sin^2 N\varphi}{1+F \sin^2\varphi}=\frac{2}{c}\cdot\frac{1+F_N \sin^2 N\varphi/2}{1+F \sin^2\varphi/2}$$

or

$$c=\frac{2(1+F_N \sin^2 N\varphi/2)(1+F \sin^2\varphi)}{(1+F \sin^2\varphi/2)(2+F_N \sin^2 N\varphi+F \sin^2\varphi)} \qquad \ldots (11)$$

This equation may be solved by giving particular values to R and N and the general solution can be written as

$$\varphi=\frac{\pi}{N_e} \qquad \ldots (12)$$

N_e being the effective number of beams.

The resolving power is given by the expression (Williams, 1950):

$$\frac{\lambda}{d\lambda}=N_e\left(n-\frac{2t}{\cos r}\frac{d\mu}{d\lambda}\right) \qquad \ldots (13)$$

Tables II and III record the values of c for different values of N_e, for a particular N value ($=50$), for $R=.9$ and $.8$ respectively. When $N=50$, $R=.9$, the value of $F_N=.021$ and $F=360$ and when $R=.8$, $F_N=.0001$ and $F=80$.

TABLE II

$R=.9$, $N=50$

N_e	40	35	30	25	20	15	10
φ	4°30′	5°8′34″	6°	7°12′	9°	12°	18°
c	.9961	.9315	.8392	.7188	.5693	.3880	.2025

TABLE III

$R=.8$, $N=50$

N_e	20	18	15	12	10	8	5
φ	9°	10°	12°	15°	18°	22°30	36°
c	1.0	.9623	.8718	.7313	.6063	.4583	.2237

In figure 2- N_e has been plotted against c, similar other graphs and tables can be prepared for different N and R values.

Considering the absorption by the material of the Lummer-Gehrcke plate the intensity expression becomes (Sodha, 1952)

$$I = I_0 \frac{[\{1-(RM)^N\}^2 + 4(RM)^N \sin^2 N\varphi/2](1-RM)}{(1-RM)^2 + 4RM \sin^2 \varphi/2} \tag{14}$$

where $M = e^{-kt \sec r}$, k being intensity absorption coefficient. In Sodha's original note there was no $(1-RM)$ term in the numerator since he neglected the initial reflection. This expression differs from (8) only in having (RM) instead of R and hence assigning particular values also for M, $N_e - c$ graphs can be drawn.

TRANSMISSION ECHELON

The expression for intensity in case of the Michelson echelon is given by (Sodha, 1953*b*)

$$I = I_0 \frac{(1-M^N)^2 + 4M^N \sin^2 N\varphi/2}{(1-M)^2 + 4M \sin^2 \varphi/2} \tag{15}$$

The expression (15) differs from (8) only in having M instead of R, hence Tables II and III and figure 2 may be used in this case also by replacing R by M.

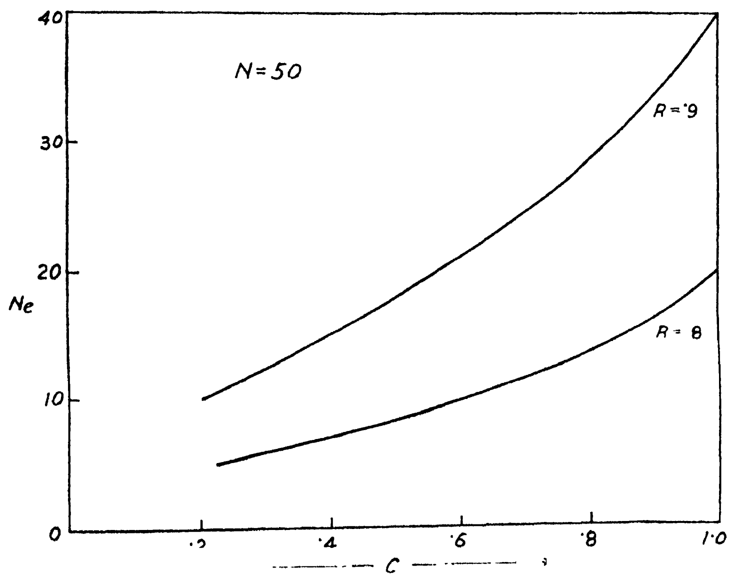

Fig. 2. Dependence of N_e on c

ACKNOWLEDGMENT

The author is grateful to Dr. K. Majumdar for his guidance in the work. He also thanks Dr. D. Sharma and Mr. M.S. Sodha for helpful discussions.

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